Calculus AB Summer Packet Mandatory

Please review and complete the following packet prior to the first day of class in Fall of the 2023 - 2024 school year. The packets will only be collected on day one. The precalculus concepts, ideas and problems are specifically designed to prepare you for the rigors of Calculus on day. The expectation is that you will spend a reasonable amount of time reflecting on these concepts that you have previously learned in Precalculus, College Algebra or trigonometry. Success in Calculus is highly correlated to the degree of a student's preparation. Have a wonderful summer. Mr. Major.

Avon High School **Name** _____
AP Calculus AB
Summer Review Packet **Score** _____ **Period** _____

Topic A: Functions

1.) If $f(x)=4x-x^2$, find:

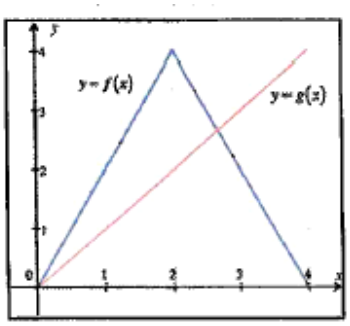
a.) $f(4)-f(-4)$ b.) $\sqrt{f\left(\frac{3}{2}\right)}$ c.) $\frac{f(x+h)-f(x)}{2h}$

2.) If $V(r)=\frac{4}{3}\pi r^3$, find:

a.) $V\left(\frac{3}{4}\right)$ b.) $V(r+1)-V(r-1)$ c.) $\frac{V(2r)}{V(r)}$

3.) If $f(x)$ and $g(x)$ are given in the graph, find:

a.) $(f-g)(3)$ b.) $f(g(3))$



4.) If $f(x)=\begin{cases}-x, & x<0\\ x^2-1, & 0\le x<2\\ \sqrt{x+2}-2, & x\ge 2\end{cases}$, find:

a.) $f(0)-f(2)$ b.) $\sqrt{5-f(-4)}$ c.) $f(f(3))$

Topic B: Domain and Range

Find the domain of the following functions using interval notation:

1.) $f(x)=3$

2.) $y=x^3-x^2+x$

3.) $y=\dfrac{x^3-x^2+x}{x}$

4.) $y=\dfrac{x-4}{x^2-16}$

5.) $f(x)=\dfrac{1}{4x^2-4x-3}$

6.) $y=\sqrt{2x-9}$

7.) $y=\log(x-10)$

8.) $y=\dfrac{\sqrt{2x+14}}{x^2-49}$

Find the range of the following functions:

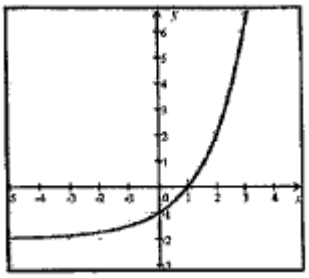
9.) $y=x^4+x^2-1$

10.) $y=100^x$

11.) $y=\sqrt{x^2+1}+1$

Find the domain and range of the following functions using interval notation.

12.)



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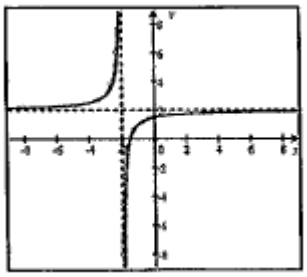

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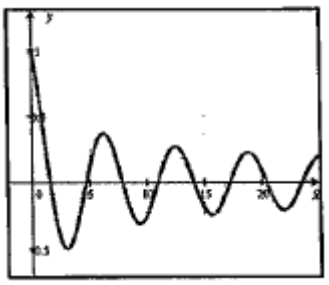
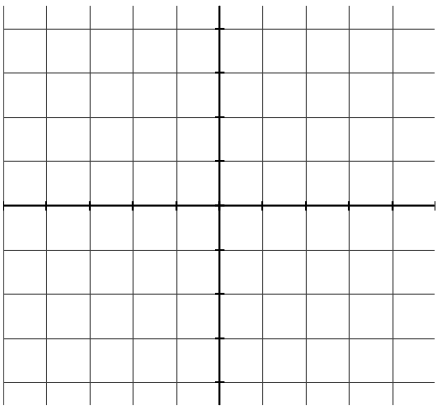


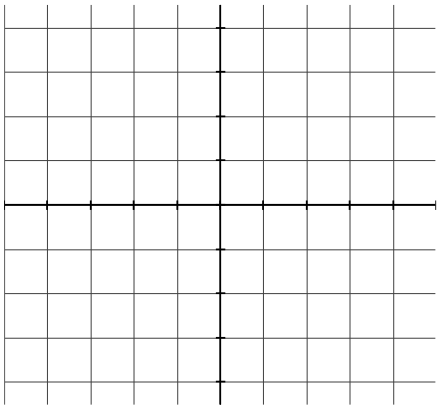
Topic C: Graphs of Common Functions

Sketch each of the following as accurately as possible. **You will need to be VERY familiar with each of these graphs throughout the year**. You may use a graphing calculator for some of them if you have access to one over the summer. If you plan on renting a TI-Nspire and thus do not have one for the summer, I strongly recommend you use try www.desmos.com. There is an app for Desmos as well that is free that you can install on your phones. Again, these are VERY important graphs to know. Be very accurate with regards to "open circles" and "closed circles" as those features may not be revealed on a graphing utility. For students who have not taken Trigonometry yet, do your best with #'s 9-14.

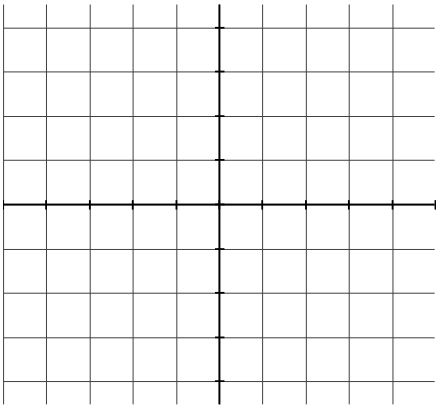
1. $y = x$



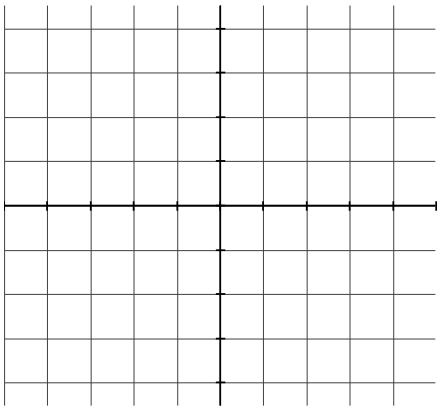
2. $y = x^2$



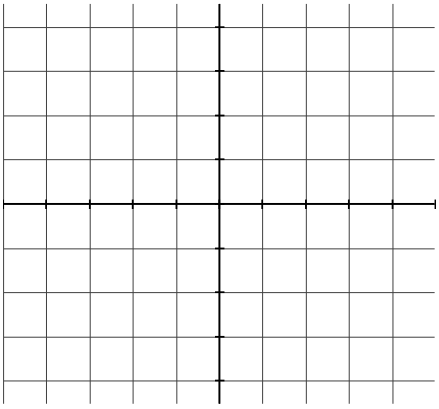
3. $y = x^3$



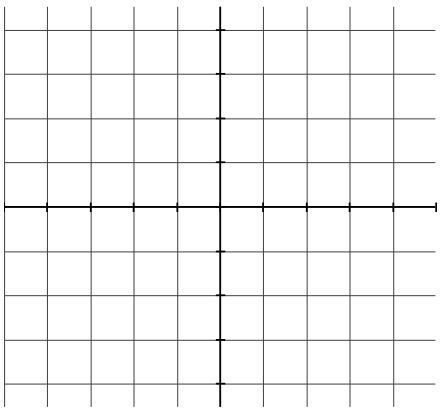
4. $y = \sqrt{x}$



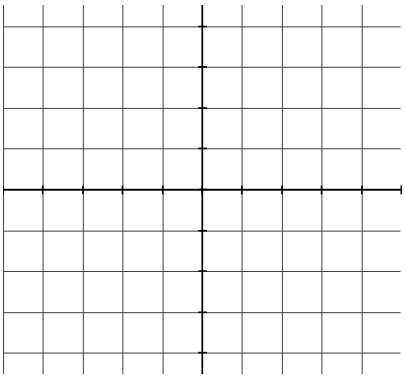
5. $y = |x|$



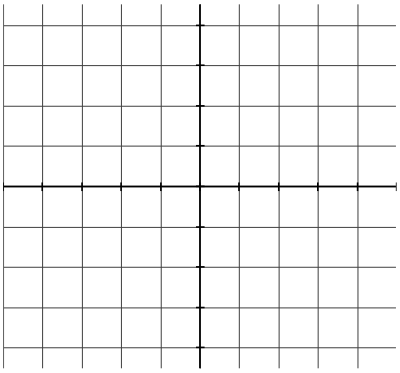
6. $y = \dfrac{|x|}{x}$



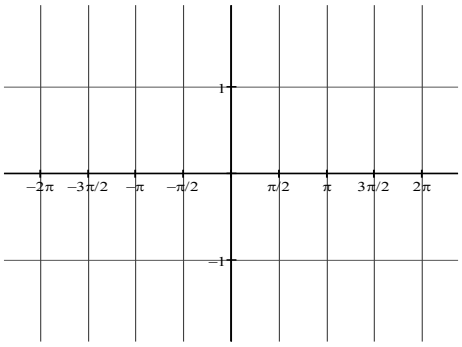
7. $y = x^{1/3}$



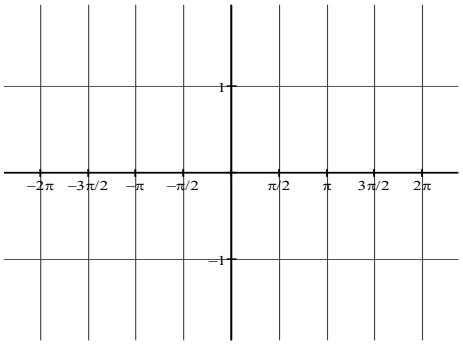
8. $y = x^{2/3}$



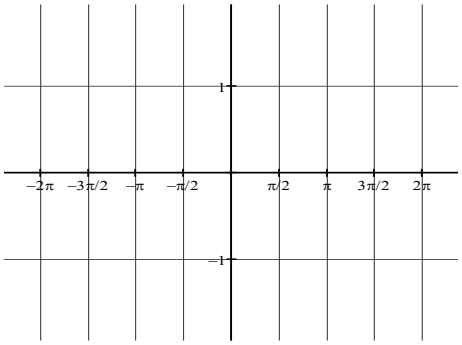
9. $y = \sin x$



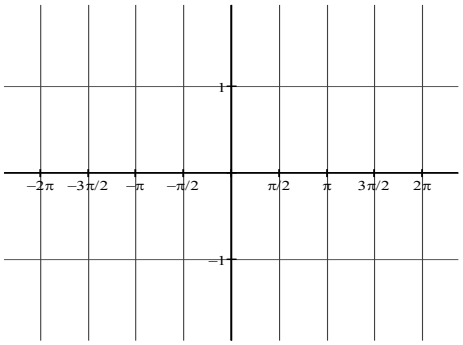
10. $y = \cos x$



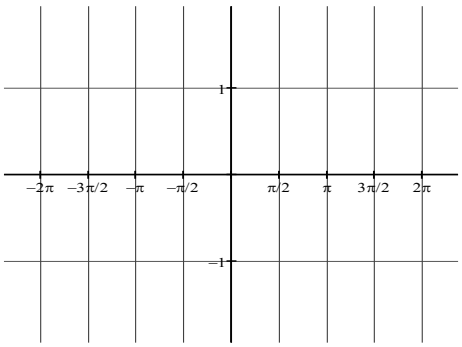
11. $y = \tan x$



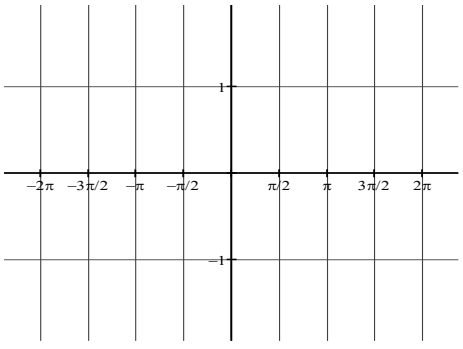
12. $y = \cot x$



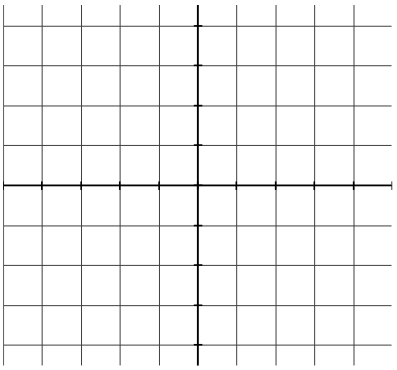
13. $y = \sec x$



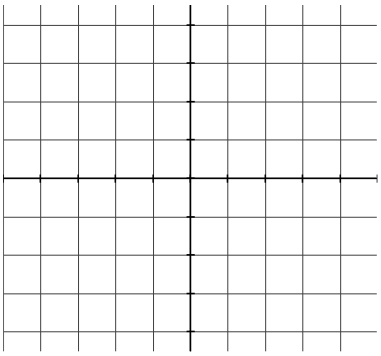
14. $y = \csc x$



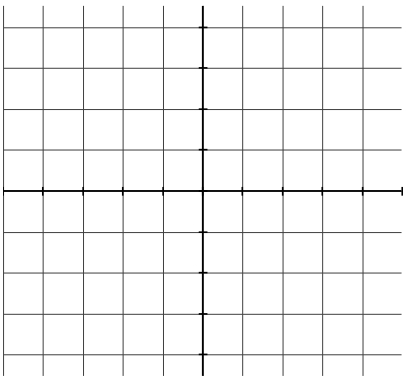
15. $y = e^x$



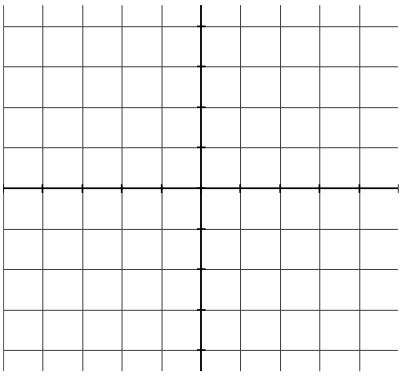
16. $y = \ln x$



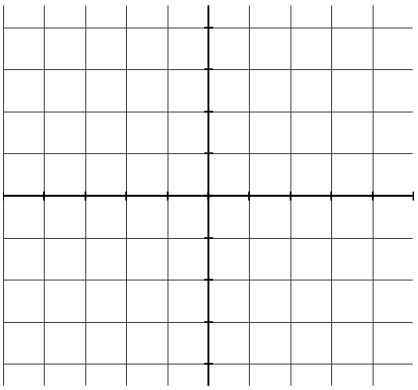
17. $y = \dfrac{1}{x}$



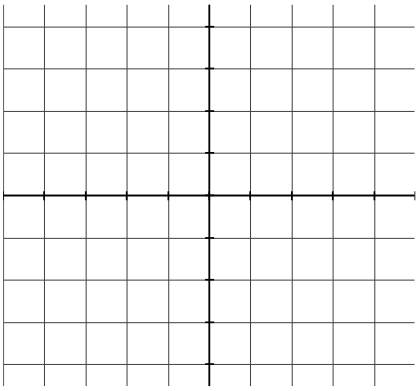
18. $y = \sqrt{x}$



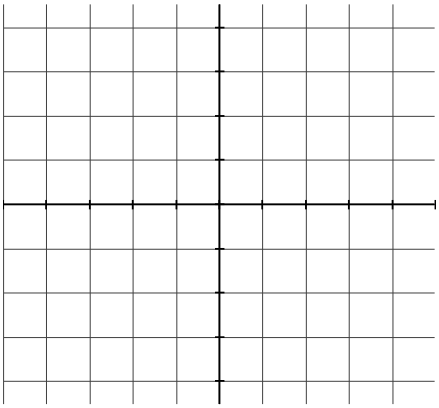
19. $y = \dfrac{1}{x^2}$



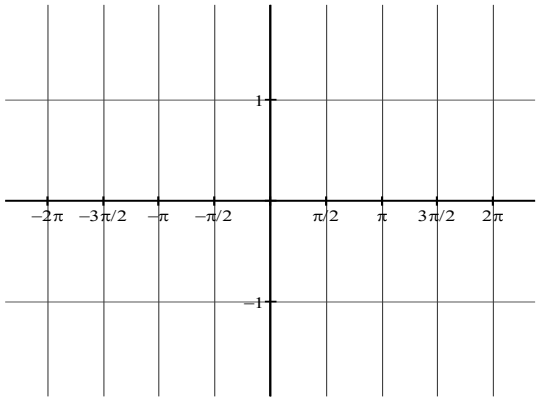
20. $y = 2^x$



21. $y = \sqrt{4 - x^2}$



22. $y = \dfrac{\sin x}{x}$



Topic D: Even/Odd Functions and Symmetry

Show work to determine if the relation is even, odd, or neither. You may want to research how to determine evenness and oddness.

1.) $f(x)=7$

2.) $f(x)=2x^2-4x$

3.) $f(x)=-3x^3-2x$

4.) $f(x)=\sqrt{x+1}$

5.) $f(x)=\sqrt{x^2+1}$

6.) $f(x)=|8x|$

Show work to determine if the graphs of these equations are symmetric to the *x*-axis, *y*-axis, or the origin.

7.) $4x=1$

8.) $y^2=2x^4+6$

9.) $3x^2=4y^3$

10.) $x=|y|$

11.) $|x|=|y|$

12.) $|x|=y^2+2y+1$

Topic E: Function Transformations

If $f(x)=x^2-1$, describe in words what the following would do to the graph of $f(x)$:

1.) $f(x)-4$

2.) $f(x-4)$

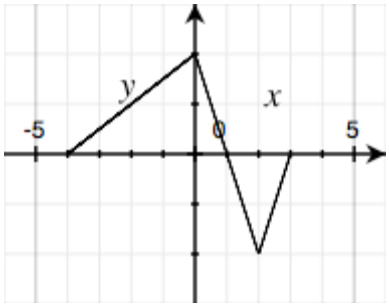
3.) $-f(x+2)$

4.) $5f(x)+3$

5.) $f(2x)$

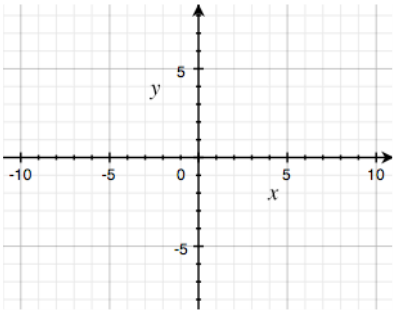
6.) $|f(x)|$

Here is a graph of $y=f(x)$:

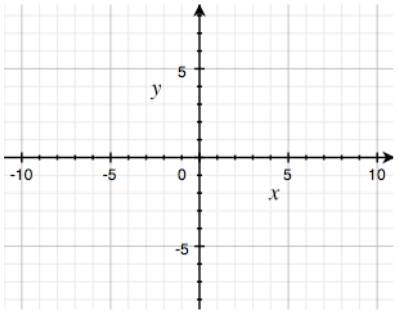


Sketch the following graphs:

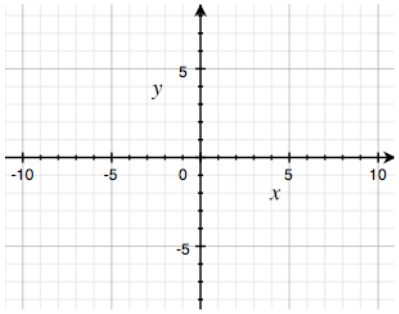
7.) $y=2f(x)$



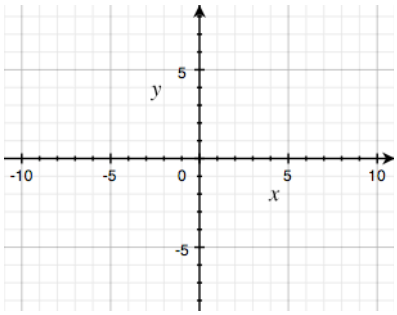
8.) $y=-f(x)$



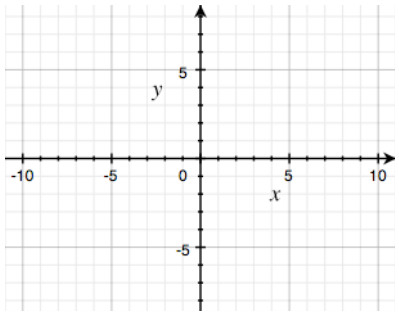
9.) $y=f(x-1)$



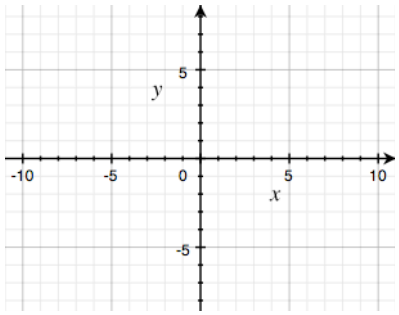
10.) $y=f(x)+2$



11.) $y=|f(x)|$



12.) $y=f(|x|)$



Topic F: Special Factorization

Factor completely.

1.) x^3+8

2.) x^3-8

3.) $27x^3-125y^3$

4.) x^4+11x^2-80

5.) $ac+cd-ab-bd$

6.) $2x^2+50y^2-20xy$

7.) $x^2+12x+36-9y^2$

8.) $x^3-xy^2+x^2y-y^3$

9.) $(x-3)^2(2x+1)^3+(x-3)^3(2x+1)^2$

Topic G: Linear Functions

1.) Find the equation of the line in point-slope form, with the given slope, passing through the given point.

a.) $m=-7,\ (-3,-7)$ b.) $m=-\frac{1}{2},\ (2,-8)$ c.) $m=\frac{2}{3},\ \left(-6,\frac{1}{3}\right)$

2.) Find the equation of the line in point-slope form, passing through the given points.

a.) $(-3,6),\ (-1,2)$ b.) $(-7,1),\ (3,-4)$ c.) $\left(-2,\frac{2}{3}\right),\ \left(\frac{1}{2},1\right)$

3.) Find the equations of the lines through the given point that are a.) parallel and b.) normal to the given line.

a.) $(5,-3),\ x+y=4$ b.) $(-6,2),\ 5x+2y=7$ c.) $(-3,-4),\ y=-2$

4.) Find the equation of the line in general form, containing the point $(4,-2)$ and parallel to the line containing the points $(-1,4)$ and $(2,3)$.

5.) Find k if the lines $3x-5y=9$ and $2x+ky=11$ are a.) parallel and b.) perpendicular.

Topic H: Solving Quadratic and Polynomial Equations

Solve each equation for x over the real number system.

1.) $x^2 + 7x - 18 = 0$

2.) $x^2 + x + \frac{1}{4} = 0$

3.) $2x^2 - 72 = 0$

4.) $12x^2 - 5x = 2$

5.) $20x^2 - 56x + 15 = 0$

6.) $81x^2 + 72x + 16 = 0$

7.) $x + \frac{1}{x} = \frac{17}{4}$

8.) $x^3 - 5x^2 + 5x - 25 = 0$

9.) $2x^4 - 15x^3 + 18x^2 = 0$

10.) If $y = x^2 + kx - k$, for what values of k will the quadratic have two real solutions?

Topic I: Asymptotes

For each function, find the equations of both the vertical asymptote(s) and horizontal asymptote (if it exists) and the location of any holes.

1.) $y = \dfrac{x-1}{x+5}$

2.) $y = \dfrac{8}{x^2}$

3.) $y = \dfrac{2x+16}{x+8}$

4.) $y = \dfrac{2x^2+6x}{x^2+5x+6}$

5.) $y = \dfrac{x}{x^2-25}$

6.) $y = \dfrac{x^2-5}{2x^2-12}$

7.) $y = \dfrac{x^3}{x^2+4}$

8.) $y = \dfrac{x^3+4x}{x^3-2x^2+4x-8}$

9.) $y = \dfrac{10x+20}{x^3-2x^2-4x+8}$

10.) $y = \dfrac{1}{x} - \dfrac{x}{x+2}$ (Hint: Express with a common denominator)

Topic J: Negative and Fractional Exponents

Simplify and write with positive exponents.

1.) $-12^2 x^{-5}$

2.) $\left(-12x^5\right)^{-2}$

3.) $\left(4x^{-1}\right)^{-1}$

4.) $\left(\frac{-4}{x^4}\right)^{-3}$

5.) $\left(\frac{5x^3}{y^2}\right)^{-3}$

6.) $\left(x^3-1\right)^{-2}$

7.) $\left(121x^8\right)^{1/2}$

8.) $\left(8x^2\right)^{-4/3}$

9.) $\left(-32x^{-5}\right)^{-3/5}$

10.) $\frac{1}{4}\left(16x^2\right)^{-3/4}\left(32x\right)$

11.) $\frac{\left(x^2-1\right)^{-1/2}}{\left(x^2+1\right)^{1/2}}$

12.) $\left(x^{-2}+2^{-2}\right)^{-1}$

Topic K: Complex Fractions

Eliminate the complex fractions:

1.) $\dfrac{\frac{5}{8}}{-\frac{2}{3}}$

2.) $\dfrac{4-\frac{2}{9}}{3+\frac{4}{3}}$

3.) $\dfrac{2+\frac{7}{2}+\frac{3}{5}}{5-\frac{3}{4}}$

4.) $\dfrac{x-\frac{1}{x}}{x+\frac{1}{x}}$

5.) $\dfrac{1+x^{-1}}{1-x^{-2}}$

6.) $\dfrac{x^{-1}+y^{-1}}{x+y}$

7.) $\dfrac{x^{-2}+x^{-1}+1}{x^{-2}-x}$

8.) $\dfrac{\frac{1}{3}(3x-4)^{-3/4}}{-\frac{3}{4}}$

9.) $\dfrac{2x(2x-1)^{1/2}-2x^2(2x-1)^{-1/2}}{(2x-1)}$

Topic L: Inverses

Find the inverse of each of the following functions and use a graphing utility (TI-Nspire or Desmos) to show graphically that its inverse is a function.

1.) $2x-6y=1$

2.) $y=ax+b$

3.) $y=9-x^2,\ x\geq 0$

4.) $y=\sqrt{1-x^3}$

5.) $y=\dfrac{9}{x}$

6.) $y=\dfrac{2x+1}{3-2x}$

Find the inverse of each of the following functions and show that $f\left(f^{-1}(x)\right)=x$

7.) $f(x)=\dfrac{1}{2}x-\dfrac{4}{5}$

8.) $f(x)=x^2-4$

9.) $f(x)=\dfrac{x^2}{x^2+1}$

10.) Without finding the inverse, find the domain and range of the inverse to $f(x)=\dfrac{\sqrt{x+1}}{x^2}$

Topic M: Adding Fractions and Solving Rational Equations

1.) Combine the following fractions:

a.) $\frac{2}{3}-\frac{1}{x}$

b.) $\frac{1}{x-3}+\frac{1}{x+3}$

c.) $\frac{5}{2x}-\frac{5}{3x+15}$

d.) $\frac{2x-1}{x-1}-\frac{3x}{2x+1}$

2.) Solve the equation for x.

a.) $\frac{2}{3}-\frac{1}{x}=\frac{5}{6}$

b.) $\frac{1}{x-3}+\frac{1}{x+3}=\frac{10}{x^2-9}$

c.) $\frac{5}{2x}-\frac{5}{3(x+5)}=\frac{5}{x}$

d.) $\frac{2x-1}{x-1}-\frac{3x}{2x+1}=\frac{x^2+11}{2x^2-x-1}$

Topic N: Absolute Value Equations

Solve the following equations:

1.) $4|x+8|=20$

2.) $|1-7x|=13$

3.) $|8+2x|+2x=40$

4.) $|4x-5|+5x+2=0$

5.) $|x^2-2x-1|=7$

6.) $|12-x|=x^2-12x$

Topic O: Solving Inequalities

Solve the following inequalities:

1.) $5(x-3) \le 8(x+5)$

2.) $4 - \frac{5x}{3} > -\left(2x + \frac{1}{2}\right)$

3.) $\frac{3}{4} > x + 1 > \frac{1}{2}$

4.) $x + 7 \ge |5 - 3x|$

5.) $(x+2)^2 < 25$

6.) $x^3 < 4x^2$

7.) $\frac{5}{x-6} \ge \frac{1}{x+2}$

8.) Find the domain of: $\sqrt{\frac{x^2 - x - 6}{x - 4}}$

Topic P: Exponential Functions and Logarithms

Simplify the following:

1.) $\log_2 \frac{1}{4}$

2.) $\log_8 4$

3.) $\ln \frac{1}{\sqrt[3]{e^2}}$

4.) $5^{\log_5 40}$

5.) $e^{\ln 12}$

6.) $\log_{12} 2 + \log_{12} 9 + \log_{12} 8$

7.) $\log_2 \frac{2}{3} + \log_2 \frac{3}{32}$

8.) $\log_{\frac{1}{3}} \frac{4}{3} - \log_{\frac{1}{3}} 12$

9.) $\log_3 \left(\sqrt{3}\right)^5$

Solve the following:

10.) $\log_5 (3x-8) = 2$

11.) $\log_9 \left(x^2 - x + 3\right) = \frac{1}{2}$

12.) $\log(x-3) + \log 5 = 2$

13.) $\log_2 (x-1) + \log_2 (x+3) = 5$

14.) $\log_5 (x+3) - \log_5 x = 2$

15.) $\ln x^3 - \ln x^2 = \frac{1}{2}$

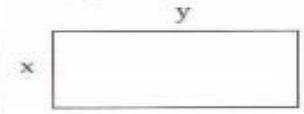
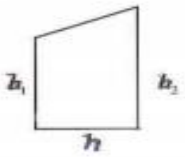
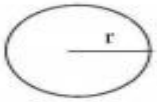
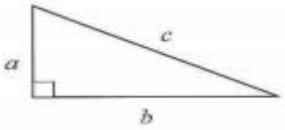
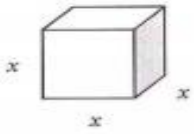
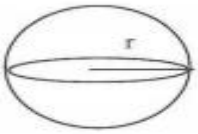
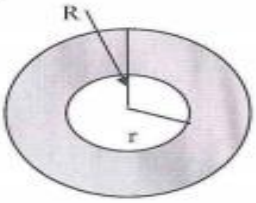
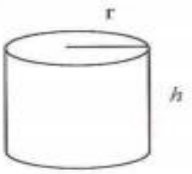
16.) $3^{x-2} = 18$

17.) $e^{3x+1} = 10$

18.) $8^x = 5^{2x-1}$

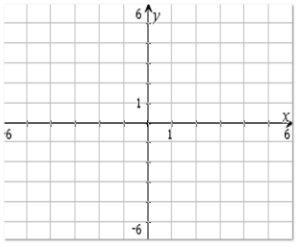
Topic Q: Geometry

1.) You will use each of the following forumlas in AP Calculus AB. Complete each of the following.

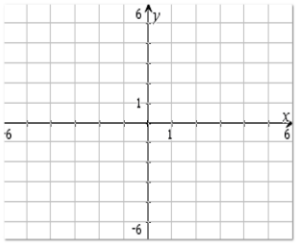
Square  Perimeter = _____ Area = _____	**Rectangle**  Perimeter = _____ Area = _____	**Trapezoid**  Area = _____
Circle  Circumference = _____ Area = _____	**Triangle**  Pythagorean Theorem (only good for right triangles) = _____ Area (of any triangle) = _____	**Cube**  Volume = _____ Surface Area = _____
Sphere  Volume = _____	**"Washer"**  Area of the shaded region = _____	**Cylinder**  Volume = _____

Find the area between the x-axis and $f(x)$ from $x = 0$ to $x = 5$. Sketch the region to verify.

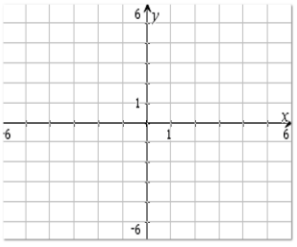
2.) $f(x) = 4$



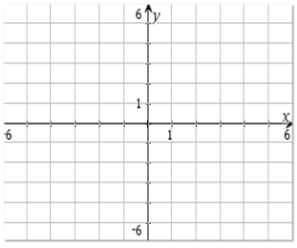
3.) $f(x) = x$



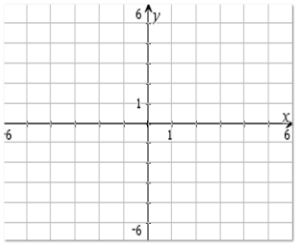
4.) $f(x) = x + 3$



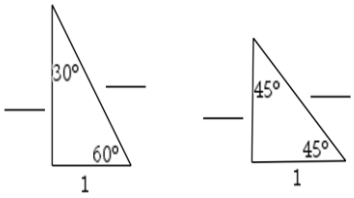
5.) $f(x) = \sqrt{9 - x^2}$



6.) $f(x) = \begin{cases} x+1, & x \le 2 \\ 5-x, & x > 2 \end{cases}$



7.) Fill in the four blanks.



Topic R: Basic Right Angle Trigonometry

(Some ACE AP Calculus students will have to complete this portion of the packet after the school year begins.)

Solve the following:

If point P is on the terminal side of θ, find all 6 trigonometric functions of θ. (Answers need not be rationalized.)

1.) $P(-2,4)$

2.) $P(\sqrt{5},-2)$

3.) If $\cos\theta = -\frac{5}{13}$, in quadrant II, find $\sin\theta$ and $\tan\theta$.

4.) If $\cot\theta = \frac{2\sqrt{10}}{3}$, in quadrant III, find $\sin\theta$ and $\cos\theta$.

5.) State the quadrant in which each of the following is true.

a.) $\sin\theta > 0$ and $\cos\theta < 0$

b.) $\csc\theta < 0$ and $\cot\theta > 0$

c.) $\tan\theta > 0$ and $\sec\theta < 0$

Topic S: Special Angles

(Some ACE AP Calculus students will have to complete this portion of the packet after the school year begins.)

Evaluate each of the following.

1.) $\sin^2 120° + \cos^2 120°$

2.) $2\tan^2 300° + 3\sin^2 150° - \cos^2 180°$

3.) $\cot^2 135° - \sin^2 210° + 5\cos^2 225°$

4.) $\cot(-30°) + 3\tan 600° - \csc(-450°)$

5.) $\left(\cos\frac{2\pi}{3} - \tan\frac{3\pi}{4}\right)^2$

6.) $\left(\sin\frac{11\pi}{6} - \tan\frac{5\pi}{6}\right)\left(\sin\frac{11\pi}{6} + \tan\frac{5\pi}{6}\right)$

Determine whether each of the following statements is true or false.

7.) $\sin\frac{\pi}{6} + \sin\frac{\pi}{3} = \sin\left(\frac{\pi}{6} + \frac{\pi}{3}\right)$

8.) $\dfrac{\cos\frac{5\pi}{3} + 1}{\tan^2\frac{5\pi}{3}} = \dfrac{\cos\frac{5\pi}{3}}{\sec\frac{5\pi}{3} - 1}$

9.) $2\left(\frac{3\pi}{2} + \sin\frac{3\pi}{2}\right)\left(1 + \cos\frac{3\pi}{2}\right) > 0$

10.) $\dfrac{\cos^3\frac{4\pi}{3} + \sin\frac{4\pi}{3}}{\cos^2\frac{4\pi}{3}} > 0$

Topic T: Trigonometric Identities

(Some ACE AP Calculus students will have to complete this portion of the packet after the school year begins.)

Verify the following identities:

1.) $(1+\sin x)(1-\sin x)=\cos^2 x$

2.) $\sec^2 x+3=\tan^2 x+4$

3.) $\dfrac{1-\sec x}{1-\cos x}=-\sec x$

4.) $\dfrac{1}{1+\tan x}+\dfrac{1}{1+\cot x}=1$

5.) $\csc(2x)=\dfrac{\csc x}{2\cos x}$

6.) $\dfrac{\cos(3x)}{\cos x}=1-4\sin^2 x$

Topic U: Solving Trigonometric Equations

(Some ACE AP Calculus students will have to complete this portion of the packet after the school year begins.)

Solve each equation on the interval $[0, 2\pi)$. Do not use a calculator.

1.) $\sin^2 x = \sin x$

2.) $3\tan^3 x = \tan x$

3.) $\sin^2 x = 3\cos^2 x$

4.) $\cos x + \sin x \tan x = 2$

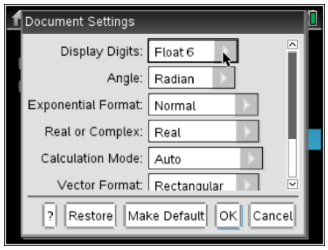
5.) $\sin x = \cos x$

6.) $2\cos^2 x + \sin x - 1 = 0$

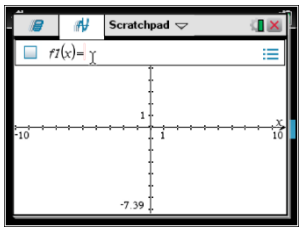
Topic V. Graphical Solutions to Equations and Inequalities

You have a shiny new TI-Nspire graphing calculator. So when are we going to use it? So far, no mention has been made of it. Yet, a graphing calculator is a tool that is required on the AP Calculus exam. For about 25% of the exam, a calculator is permitted. So it is vital you are comfortable using it.

There are several settings of the calculator you should make. First, so you don't get into rounding difficulties, it is suggested you set your calculator to show at least three decimal places (and preferably more). This is sandard on the AP Calculs exam, so it's best we start getting used to it. To do this, press the [on] button, followed by 5 Settings, then 2: Document Settings… Be sure the Display Digits option is set to Float 6. This will ensure that you always see 6 digits across the screen. (There may be times that this can be a problem – i.e. when you have a decimal answer with four or more digits to the left of the decimal. We'll deal with this later.) Also, be sure that your calculator's Angle Setting is in Radian mode throughout the year. To make these changes "stick" select Make Default at the bottom.



You must know how to graph functions on your TI-Nspire. The best way to graph a function is to press the Calculator [ ] key (located in the upper left corner of your handheld just below [esc]) twice on the left side. Notice how each time you press this button, your screen toggles between a calculator page and a graphing page. While on a graphing page, select e to bring up the function entry line. Next input the expression you wish to graph and press Enter.



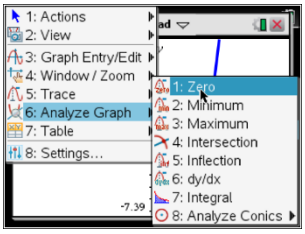
How to find zeros (solutions or *x*-intercepts) of a function.

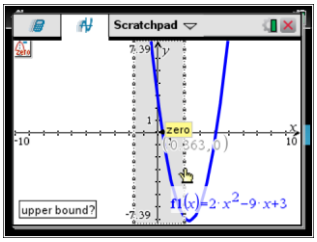
Step 1: Enter the left side of the equation that's alreay set equal to zero (for example, $2x^2 - 9x + 3 = 0$) into the $f1(x)$ entry line.

Step 2: Select [menu] followed by 6: Analyze Graph and 1: Zero

Step 3: Move the cursor to a position you feel is left of the zero you wish to find first and press Enter. Then move the cursor to the right of the desired zero you wish to find and press Enter. You will notice that the ordered pair for the zero will show up automatically once it falls within the range of your lower and upper bound.

You can find relative Maximum and Minimum values in a similar way.





How to find the intersection of two functions.

Step 1: Enter each side of the equation (for example, $x^3 = 2x - 3$ into the $f1(x)$ and $f2(x)$ entry lines.

Step 2: Select [menu] followed by 6: Analyze Graph and 4: Intersection

Step 3: Repeat Step 3 above.

This problem could also be solved by setting the above equation equal to zero, and using the following procedure "[menu] 6: Analyze Graph and 1: Zero" instead.

Note: You can always move things around on your screen and place them in positions that may make them easier to read. To do this, rub your finger over the Touchpad until the cursor appears. Move the cursor to the item (i.e. intersection ordered pair from the example above) you want to move. An open hand should appear.

Press Enter to close the hand. By rubbing the Touchpad, the ordered pair should move. Press Enter when you have found a suitable place to put it.

We will learn much more about the functionality of this calculator in the first several days of class. While many of these problems on this page can be done with other graphing utilities (desmos, etc.), I highly recommned that you wait to do these problems when you have a TI-Nspire.

Topic V: Using the TI-Nspire (Continued)

Use your TI-Nspire to find the zeros of each of the following functions. Make sure each equation is set equal to zero first.

1.) $3x^3 - x - 5 = 0$

2.) $2x^2 - 1 = 2^x$

3.) $2\ln(x+1) = 5\cos x$ on $[0, 2\pi)$

Use your TI-Nspire to find the solution (intersection) of the given system of equations.

4.) $\begin{cases} f(x) = x^4 - 6.5x^2 + 6x + 2 \\ g(x) = 1 + x + e^{x^2 - 2x} \end{cases}$

Use your TI-Nspire to find both a relative maximum and a relative minimum point of the given function.

5.) $h(x) = 2x^5 - 3x^4 + x - 4$